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The Educational Institute

Leader in CBSE Coaching

CODE : AVTE / M / XII / 17–18 / PP1

Name: ..... **Group:** .....

Board Roll No.

- Please check that this question paper contains **4 Printed Pages**.
- Code number given on the right–hand side of the question paper should be written on the title page of the answer–book by the candidate.
- Please check that this question paper contains **29 Questions**.

MATHEMATICS (XII)

[Time allowed: 3 hours] **[Maximum marks: 100]**

General Instructions :

(i) ***All questions are compulsory.***

(ii) *The question paper consists of **29 questions** divided into four **sections A, B, C and D**. **Section A** comprises of 04 questions of **one mark each**, **Section B** comprises of 08 questions of **two marks each**, **Section C** comprises of 11 questions of **four marks each** and **Section D** comprises of 06 questions of **6 marks each**.*

(iii) *All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.*

(iv) *There is no overall choice, however internal choice is provided in **03 questions** of **four marks** and **03 questions** of **six marks**. You have to attempt only one of the alternatives in all such questions.*

(v) ***Use of calculator is not permitted.***

SECTION – A

1. Write the vector equation of the line $\frac{x+1}{-3} = \frac{2-y}{-6}$, $z = 3$.

2. Evaluate : $\int_0^3 \log\left(\frac{3}{x} - 1\right) dx$.

3. Evaluate : $\int e^x \left(\cot x - \text{cosec}^2 x\right) dx$.

4. Evaluate : $\sec^2(\tan^{-1} 2)$.

SECTION – B

5. For two non zero vectors $\vec{a}$ and $\vec{b}$, it is given that $\left|\vec{a} \cdot \vec{b}\right| = \sqrt{3}\left|\vec{a} \times \vec{b}\right|$. Find the angle between $\vec{a}$ and $\vec{b}$.

6. Find the point(s) of discontinuity for $f(x) = \frac{1}{x-3}$.

7. Using determinants, find the equation of the line joining the point A(–1, 1) and B(2, 3).

8. Show that $\int \left(\frac{1}{\log x} - \frac{1}{(\log x)^2} \right) dx = \frac{x}{\log x} + c$.

9. Let $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$, $P(A \cap B) = \frac{4}{13}$, find P(A'/B).

10. Evaluate : $\int \frac{\sin x \, dx}{\sqrt{\sin^2 x - \sin^2 a}}$.

11. Evaluate : $\int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$.

12. Find the slope of the tangent for $f(x) = e^{2x}$ at (0, 1).

SECTION – C

13. Evaluate : $\int \frac{dx}{\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x}$.

OR

Prove that : $\int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx = \pi^2$.

14. Show that the differential equation $(x - y)\frac{dy}{dx} = x + 2y$ is homogeneous and hence solve it.

15. If $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$, find the value of x.

16. To promote yoga, an organization tried to generate awareness through

(i) SMS (ii) letters (iii) announcements.

The cost of each made per attempt is

(i) ` 6 (ii) ` 30 (iii) ` 40

The total number of attempts made in three localities A, B and C are given below :

	A	B	C
(i)	400	300	500
(ii)	100	150	200
(iii)	200	300	100

Find the cost incurred by the organisation for three localities separately, using matrices. Write two lines about the benefit of Yoga.

17. Evaluate : $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$.

OR

Evaluate : $\int_0^1 \sin^{-1} \frac{2x}{1 + x^2}\, dx$.

18. Find the intervals in which the function $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is strictly increasing or strictly decreasing.

19. In a tetrahedron die, four faces are numbered 1 to 4. The probability of getting even number is twice the probability of getting an odd number. Find the probability distribution for perfect squares if the same die is rolled thrice.

20. Using vectors find the area of the triangle ABC with vertices A(1, 2, 3), B(2, –1, 4) and C(4, 5, –1).

OR

Using vectors prove that parallelogram on the same base and between the same parallels are equal in area.

21. Let f : R $\to$ R defined by $f(x) = \begin{cases} 1 & ; \ x > 0 \\ 0 & ; \ x = 0 \\ -1 & ; \ x < 0 \end{cases}$ and g : R $\to$ R defined as g(x) = [x].

Find fog and gof in [0, 1]. Is fog = gof in [0, 1]?

22. Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing or decreasing. Also find the points at which the tangents are parallel to the x–axis.

23. Find the derivative of $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$ w.r.t. $\tan^{-1}\left(\frac{x}{a}\right)$ and comment.

SECTION – D

24. Find the direction ratio of the normal to the plane, which passes through the points (1, 0, 0) and (0, 1, 0) and makes angle $\frac{\pi}{4}$ with the plane x + y = 3. Also find the equation of plane.

OR

A line makes angles α, β, γ and δ with the diagonals of a cube, prove that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

25. Evaluate : $\int_0^{\pi/4} \frac{\sin 2x}{\sin^4 x + \cos^4 x}\, dx$.

26. Using integration find the area of the following region $\left\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\right\}$.

OR

Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.

27. Find the area of greatest rectangele that can be inscribed in an ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

OR

Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its sides. Also find the maximum volume.

28. A toy manufacturer produces two types of toys; a basic version toy A and a deluxe version toy B. Each toy of type B takes twice as long to produce as one toy of type A. The company has time to make a maximum of 2000 toys of type A per day, the supply of plastic is sufficient to produce 1500 toys per day and each type requires equal amount of it. Type B requires a fancy dress of which there are only 600 per day available. If the company makes a profit of ` 30 and ` 50 per toy, respectively on A and B, how many of each type should be produced per day in order to maximize profit? Make an LPP and solve it graphically.

29. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that A satisfies the equation $A^3 - 4A^2 - 3A + 11I = 0$. Hence find the A^{-1}.